Applied Mathematics and Mechanics
(English Edition, Vol 23, No 12, Dec 2002)

Published by Shanghai University,
Shanghai, China

Article ID: 0253-4827(2002)12-1357-10

THE BEHAVIOR OF TWO PARALLEL SYMMETRIC PERMEABLE CRACKS IN PIEZOELECTRIC MATERIALS *

ZHOU Zhen-gong (周振功), WANG Biao (王 彪)

(P O Box 2147, Center for Composite Materials and Electro-Optics Research Center,
Harbin Institute of Technology, Harbin 150001, P R China)

(Contributed by WANG Biao)

Abstract: *The behavior of two parallel symmetric cracks in piezoelectric materials under anti-plane shear loading was studied by the Schmidt method for the permeable crack face conditions. By using the Fourier transform, the problem can be solved with two pairs of dual integral equations in which the unknown variable is the jump of the diplacement across the crack surfaces. These equations were solved using the Schmidt method. The results show that the stress and the electric displacement intensity factors of cracks depend on the geometry of the crack. Contrary to the impermeable crack surface condition solution, it is found that the electric displacement intensity factors for the permeable crack surface conditions are much smaller than the results for the impermeable crack surface conditions.*

Key words: piezoelectric material; parallel crack; Schmidt method; dual-integral equation; stress and electric displacement; intensity factors

CLC number: O345.51 **Document code**: A

Introduction

It is well-known that piezoelectric materials produce an electric field when deformed and undergo deformation when subjected to an electric field. The coupling nature of piezoelectric materials has attracted wide applications in electric-mechanical and electric devices, such as electric-mechanical actuators, sensors and structures. When subjected to mechanical and electrical loads in service, these piezoelectric materials can fail prematurely due to defects, e.g. cracks, holes, etc. arising during their manufacture process. Therefore, it is of great importance to study the electro-elastic interaction and fracture behavior of piezoelectric materials, especially when multiple cracks are involved.

In the theoretical studies of crack problems, several different electric boundary conditions at the crack surfaces have been proposed by numerous researchers. For example, for the sake of

* **Received date**: 2001-07-19; **Revised date**: 2002-04-26

Foundation items: the National Natural Science Foundation for Excellent Young Investigators (19725209); the Natural Science Foundation of Heilongjiang Province; the Post Doctoral Science Foundation of Heilongjiang Province

Biography: ZHOU Zhen-gong (1963 –), Professor, Doctor (E-mail: zhouzhg@hope.hit.edu.cn)

analytical simplification, the assumption that the crack surfaces are impermeable to electric fields was adopted by Deeg[1], Pak[2,3], Sosa and Pak[4], Sosa[5,6], Suo, Kuo, Barnett and Willis[7], Park and Sun[8]; Zhang and Tong[9]; Gao, Zhang and Tong[10]; Wang[11]; Narita and Shindo[12,13]; Zhou, Wang and Cao[14] and Yu and Chen[15]. In these models, the assumption of the impermeable cracks refers to the fact that the crack surfaces are free of surface charge and thus the electric displacement vanishes inside the crack. In fact, cracks in piezoelectric materials consist of vacuum, air or some other gas. This requires that the electric fields can propagate through the crack, so the electirc displacement component perpendicular to the crack surfaces should be continuous across the crack surfaces. However, due to much simpler treatment from a mathematical point of view, the impermeable crack and the conducting crack are still employed extensively in the study of the crack problems of piezoelectric materials. For the permeable crack model, Zhang and Hack[16] analyzed crack problems in piezoelectric materials. In addition, usually the conducting cracks which are filled with conducting gas or liquid are also applied to be a kind of simplified cracks models in piezoelectric materials by many researchers, such as McMeeking[17] and Suo[18]. Recently, Dunn[19], Zhang and Tong[20] and Sosa and Khutoryansky[21] avoided the common assumption of electric impermeability and utilized more accurate electric boundary conditions at the rim of an elliptical flaw to deal with anti-plane problems in piezoelectricity. They analyzed the effects of electric boundary conditions at the crack surfaces on the fracture mechanics of piezoelectric materials. It is interesting to note that very different results were obtained by changing the boundary conditions. Most recently, Soh, Fang and Lee[22] have investigated the behavior of a bi-piezoelectric ceramic layer with an interfacial crack by using the dislocation density function and the singular integral equation method for two different crack surface boundary conditions, respectively, i.e. permeable and impermeable. To our knowledge, the electro-elastic behavior of two parallel symmetric permeable cracks under anti-plane shear loading in piezoelectric materials has not been studied. Accordingly, there is a need to investigate the electro-elastic fracture problem of multi- cracks in piezoelectric materials.

In the present paper, the interaction between two parallel symmetrical cracks subjected to anti-plane shear loading in piezoelectric materials is investigated using the Schmidt method[23]. It is a simple and convenient method for solving this problem. Fourier transform is applied and a mixed boundary value problem is reduced to two pairs of dual integral equations. In solving the dual integral equations, the gaps of the crack surface displacement are expanded in a series of Jacobi polynomials. This process is quite different from that adopted in previous works (Refs.[1 - 13], [15 - 22]). The form of solution is easy to understand. Numerical examples are provide to show the effect of the geometry of the cracks upon the stress intensity factor of the cracks.

1 Formulation of the Problem

It is assumed that there are two parallel symmetric cracks of length $2l$ in piezoelectric materials as shown in Fig. 1. h is the distance between the two cracks. The piezoelectric boundary-value problem for anti-plane shear is considerably simplified if we consider only the out-of-plane displacement and the in-plane electric fields. As discussed in Soh's[22] work, since no opening displacement exists for the present anti-plane problem, the crack surfaces can be assumed to be in perfect contact. Accordingly, permeable condition will be enforced in the present study, i.e., both the electric potential and the normal electric displacement are assumed to be continuous across the crack surfaces. So the boundary conditions of the present problem are (In this paper, we just consider the perturbation field)

$$w^{(1)} = w^{(2)},\ \tau_{yz}^{(1)} = \tau_{yz}^{(2)},\ \phi^{(1)} = \phi^{(2)},\ D_y^{(1)} = D_y^{(2)},\ y = h \qquad |x| > l, \tag{1}$$

$$w^{(2)} = w^{(3)},\ \tau_{yz}^{(2)} = \tau_{yz}^{(3)},\ \phi^{(2)} = \phi^{(3)},\ D_y^{(2)} = D_y^{(3)},\ y = 0 \qquad |x| > l, \tag{2}$$

$$\tau_{yz}^{(1)} = \tau_{yz}^{(2)} = -\tau_0,\ \phi^{(1)} = \phi^{(2)},\ D_y^{(1)} = D_y^{(2)},\ y = h \qquad |x| \leqslant l, \tag{3}$$

$$\tau_{yz}^{(2)} = \tau_{yz}^{(3)} = -\tau_0,\ \phi^{(2)} = \phi^{(3)},\ D_y^{(2)} = D_y^{(3)},\ y = 0 \qquad |x| \leqslant l, \tag{4}$$

$$w^{(1)} = w^{(2)} = w^{(3)} = 0 \qquad \text{for } (x^2 + y^2)^{1/2} \to \infty, \tag{5}$$

where τ_{zk}, D_k $(k = x, y)$ are the anti-plane shear stress and in-plane electric displacement, respectively. w and ϕ are the mechanical displacement and the electric potential. Note that all quantities with superscript k $(k = 1,2,3)$ refer to the upper half plane 1, the layer 2 and the lower half plane 3 as in Fig.1, respectively. In this paper, we only consider that τ_0 is positive. The constitutive equations can be written as

$$\tau_{zk} = c_{44} w_{,k} + e_{15}\phi_{,k}, \tag{6}$$

$$D_k = e_{15} w_{,k} - \varepsilon_{11}\phi_{,k}, \tag{7}$$

c_{44}, e_{15}, ε_{11} are the shear modulus, piezoelectric coefficient and dielectric parameter, respectively. The anti-plane governing equations are[22]

$$c_{44}\nabla^2 w + e_{15}\nabla^2\phi = 0, \tag{8}$$

$$e_{15}\nabla^2 w - \varepsilon_{11}\nabla^2\phi = 0, \tag{9}$$

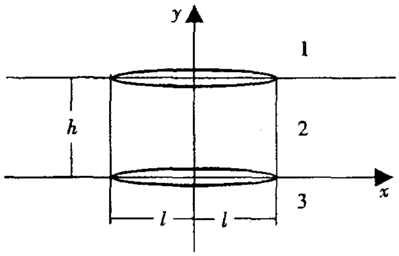

Fig.1 Two parallel symmetric cracks in a piezoelectric material

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the two-dimensional Laplace operator. Because of the assumed symmetry in geometry and loading, it is sufficient to consider the problem for $0 \leqslant x < \infty$, $0 \leqslant y < \infty$ only. A Fourier transform is applied to Eqs.(8) and (9). Assume that the solutions are

$$\begin{cases} w^{(1)}(x,y) = \dfrac{2}{\pi}\displaystyle\int_0^\infty A_1(s)e^{-sy}\cos(sx)\,ds \\ \phi^{(1)}(x,y) = \dfrac{e_{15}}{\varepsilon_{11}} w^{(1)}(x,y) + \dfrac{2}{\pi}\displaystyle\int_0^\infty B_1(s)e^{-sy}\cos(sx)\,ds \end{cases} \qquad (y \geqslant h), \tag{10}$$

$$\begin{cases} w^{(2)}(x,y) = \dfrac{2}{\pi}\displaystyle\int_0^\infty [A_2(s)e^{-sy} + B_2(s)e^{sy}]\cos(sx)\,ds \\ \phi^{(2)}(x,y) = \dfrac{e_{15}}{\varepsilon_{11}} w^{(2)}(x,y) + \dfrac{2}{\pi}\displaystyle\int_0^\infty [C_2(s)e^{-sy} + D_2(s)e^{sy}]\cos(sx)\,ds \end{cases} \qquad (h \geqslant y \geqslant 0), \tag{11}$$

$$\begin{cases} w^{(3)}(x,y) = \dfrac{2}{\pi}\displaystyle\int_0^\infty A_3(s)e^{sy}\cos(sx)\mathrm{d}s \\ \phi^{(3)}(x,y) = \dfrac{e_{15}}{\varepsilon_{11}}w^{(3)}(x,y) + \dfrac{2}{\pi}\displaystyle\int_0^\infty B_3(s)e^{sy}\cos(sx)\mathrm{d}s \end{cases} \quad (y \leqslant 0), \tag{12}$$

where $\mu = c_{44} + e_{15}^2/\varepsilon_{11}$, $A_1(s)$, $B_1(s)$, $A_2(s)$, $B_2(s)$, $C_2(s)$, $D_2(s)$, $A_3(s)$ and $B_3(s)$ are unknown functions, and a superposed bar indicates the Fourier transform throughout the paper, e.g.

$$\bar{f}(s) = \int_{-\infty}^{\infty} f(x)e^{-isx}\mathrm{d}x. \tag{13}$$

So from Eqs.(6) and (7), we have

$$\begin{cases} \tau_{yz}^{(1)}(x,y) = -\dfrac{2}{\pi}\displaystyle\int_0^\infty s[\mu A_1(s)e^{-sy} + \\ \qquad e_{15}B_1(s)e^{-sy}]\cos(sx)\mathrm{d}s \qquad (y \geqslant h), \\ D_y^{(1)}(x,y) = \dfrac{2}{\pi}\displaystyle\int_0^\infty \varepsilon_{11}sB_1(s)e^{-sy}\cos(sx)\mathrm{d}s \end{cases} \tag{14}$$

$$\begin{cases} \tau_{yz}^{(2)}(x,y) = -\dfrac{2}{\pi}\displaystyle\int_0^\infty \Big\{\mu s[A_2(s)e^{-sy} - B_2(s)e^{sy}] + \\ \qquad e_{15}s[C_2(s)e^{-sy} - D_2(s)e^{sy}]\Big\}\cos(sx)\mathrm{d}s \qquad (h \geqslant y \geqslant 0), \\ D_y^{(2)}(x,y) = \dfrac{2}{\pi}\displaystyle\int_0^\infty \varepsilon_{11}s[C_2(s)e^{-sy} - D_2(s)e^{sy}]\cos(sx)\mathrm{d}s \end{cases} \tag{15}$$

$$\begin{cases} \tau_{yz}^{(3)}(x,y) = \dfrac{2}{\pi}\displaystyle\int_0^\infty s[\mu A_3(s)e^{sy} + e_{15}B_3(s)e^{sy}]\cos(sx)\mathrm{d}s \\ D_y^{(3)}(x,y) = -\dfrac{2}{\pi}\displaystyle\int_0^\infty \varepsilon_{11}sB_3(s)e^{sy}\cos(sx)\mathrm{d}s \end{cases} \quad (y \leqslant 0). \tag{16}$$

For solving the problem, the gap functions of the crack surface displacements and the electric potentials are defined as follows:

$$f_1(x) = w^{(1)}(x,h^+) - w^{(2)}(x,h^-), \tag{17}$$

$$f_{\phi 1}(x) = \phi^{(1)}(x,h^+) - \phi^{(2)}(x,h^-), \tag{18}$$

$$f_2(x) = w^{(2)}(x,0^+) - w^{(3)}(x,0^-), \tag{19}$$

$$f_{\phi 2}(x) = \phi^{(2)}(x,0^+) - \phi^{(3)}(x,0^-). \tag{20}$$

Substituting Eqs.(10) – (12) into Eqs.(17) – (20), and applying the Fourier transform and the boundary conditions, it can be obtained

$$\bar{f}_1(s) = [A_1(s) - A_2(s)]e^{-sh} - B_2(s)e^{sh}, \tag{21}$$

$$\bar{f}_{\phi 1}(s) = \frac{e_{15}}{\varepsilon_{11}}\bar{f}_1(s) + [B_1(s) - C_2(s)]e^{-sh} - D_2(s)e^{sh} = 0, \tag{22}$$

$$\bar{f}_2(s) = A_2(s) + B_2(s) - A_3(s), \tag{23}$$

$$\bar{f}_{\phi 2}(s) = \frac{e_{15}}{\varepsilon_{11}}\bar{f}_2(s) + C_2(s) + D_2(s) - B_3(s) = 0. \tag{24}$$

Substituting Eqs.(14) - (16) into Eqs.(1) - (4), it can be obtained

$$\mu A_1(s)e^{-sh} + e_{15}B_1(s)e^{-sh} = \mu[A_2(s)e^{-sh} - B_2(s)e^{sh}] + e_{15}[C_2(s)e^{-sh} - D_2(s)e^{sh}], \tag{25}$$

$$[B_1(s) - C_2(s)]e^{-2sh} + D_2(s) = 0, \tag{26}$$

$$\mu[A_2(s) - B_2(s)] + e_{15}[C_2(s) - D_2(s)] = -\mu A_3(s) - e_{15}B_3(s), \tag{27}$$

$$C_2(s) - D_2(s) + B_3(s) = 0. \tag{28}$$

By solving eight Eqs.(21) - (28) with eight unknown functions $A_1(s)$, $B_1(s)$, $A_2(s)$, $B_2(s)$, $C_2(s)$, $D_2(s)$, $A_3(s)$, $B_3(s)$ and applying the boundary conditions (3) - (4), we can obtain

$$\int_0^\infty sc_{44}[\bar{f}_1(s) + e^{-sh}\bar{f}_2(s)]\cos(sx)ds = \pi\tau_0, \qquad |x| \leqslant l, \tag{29}$$

$$\int_0^\infty sc_{44}[\bar{f}_2(s) + e^{-sh}\bar{f}_1(s)]\cos(sx)ds = \pi\tau_0, \qquad |x| \leqslant l, \tag{30}$$

$$\int_0^\infty \bar{f}_1(s)\cos(sx)ds = 0, \qquad |x| > l, \tag{31}$$

$$\int_0^\infty \bar{f}_2(s)\cos(sx)ds = 0, \qquad |x| > l. \tag{32}$$

From Eqs.(29) - (32), it can be obtained

$$\begin{cases} \bar{f}_1(s) = \bar{f}_2(s) \Rightarrow f_1(x) = f_2(x), \\ \tau_{yz}^{(1)}(x,h) = \tau_{yz}^{(2)}(x,h) = \tau_{yz}^{(2)}(x,0) = \tau_{yz}^{(3)}(x,0). \end{cases} \tag{33}$$

So from Eqs.(14) - (16), it can be obtained $D_y^{(1)}(x,h) = D_y^{(2)}(x,h) = D_y^{(2)}(x,0) = D_y^{(3)}(x,0)$. To determine the unknown functions $\bar{f}_1(s)$ and $\bar{f}_2(s)$, the dual-integral equations (29) - (32) must be solved.

2 Solution of the Dual Integral Equation

The Schmidt method[23] is used to solve the dual integral equations (29) - (32). The gap functions of the crack surface displacement are represented by the following series:

$$f_1(x) = f_2(x) = \sum_{n=1}^{\infty} a_n P_{2n-2}^{(1/2,1/2)}\left(\frac{x}{l}\right)\left(1 - \frac{x^2}{l^2}\right)^{\frac{1}{2}} \qquad \text{for } -l \leqslant x \leqslant l,\ y = 0, \tag{34}$$

where a_n is unknown coefficients to be determined and $P_n^{(1/2,1/2)}(x)$ is a Jacobi polynomial[24] The Fourier transform of Eq.(34) are[25]

$$\bar{f}_1(s) = \sum_{n=1}^{\infty} a_n G_n \frac{1}{s} J_{2n-1}(sl), \quad G_n = 2\sqrt{\pi}(-1)^{n-1}\frac{\Gamma(2n - 1/2)}{(2n-2)!}, \tag{35}$$

where $\Gamma(x)$ and $J_n(x)$ are the Gamma and Bessel functions, respectively.

Substituting Eq.(35) into Eqs.(29) - (32), Eqs.(31) - (32) has been automatically satisfied, respectively. Then the Eq.(29) reduces to the form after integration with respect to x for $-l < x < l$,

$$\sum_{n=1}^{\infty} a_n G_n \int_0^{\infty} \frac{c_{44}}{s}[1 + e^{-sh}] J_{2n-1}(sl)\sin(sx)ds = \pi\tau_0 x. \tag{36}$$

From the relationship[25]

$$\int_0^{\infty} \frac{1}{s} J_n(sa)\sin(bs)ds = \begin{cases} \dfrac{\sin[n\arcsin(b/a)]}{n}, & a > b, \\ \dfrac{a^n \sin(n\pi/2)}{n[b + \sqrt{b^2 - a^2}]^n}, & b > a. \end{cases} \tag{37}$$

The semi-infinite integral in Eq. (36) can be modified as

$$\int_0^{\infty} \frac{1}{s}[1 + e^{-sh}] J_{2n-1}(sl)\sin(sx)ds =$$
$$\frac{1}{2n-1}\sin\left[(2n-1)\arcsin\left(\frac{x}{l}\right)\right] + \int_0^{\infty} \frac{1}{s} e^{-sh} J_{2n-1}(sl)\sin(sx)ds. \tag{38}$$

For a large s, the integrands of the semi-infinite integral in Eq. (38) are almost all e^{-sh}. Thus they can be evaluated directly by Filon's method[26]. Eq. (36) can now be solved for the coefficients a_n by the Schmidt method[23]. For brevity, the Eq. (36) can be rewritten as

$$\sum_{n=1}^{\infty} a_n E_n(x) = U(x), \qquad -l < x < l, \tag{39}$$

where $E_n(x)$ and $U(x)$ are known functions and the coefficients a_n are to be determined. A set of functions $P_n(x)$ which satisfy the orthogonality condition

$$\int_{-l}^{l} P_m(x) P_n(x)dx = N_n \delta_{mn}, \qquad N_n = \int_{-l}^{l} P_n^2(x)dx, \tag{40}$$

can be constructed from the function, $E_n(x)$, such that

$$P_n(x) = \sum_{i=1}^{n} \frac{M_{in}}{M_{nn}} E_i(x), \tag{41}$$

where M_{ij} is the cofactor of the element d_{ij} of $\boldsymbol{D}_n$, which is defined as

$$\boldsymbol{D}_n = \begin{pmatrix} d_{11} & d_{12} & d_{13} & \cdots & d_{1n} \\ d_{21} & d_{22} & d_{23} & \cdots & d_{2n} \\ d_{31} & d_{32} & d_{33} & \cdots & d_{3n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ d_{n1} & d_{n2} & d_{n3} & \cdots & d_{nn} \end{pmatrix}, \qquad d_{ij} = \int_{-l}^{l} E_i(x) E_j(x)dx. \tag{42}$$

Using Eqs. (39) – (42), we obtain

$$a_n = \sum_{j=n}^{\infty} q_j \frac{M_{nj}}{M_{jj}} \text{ with } q_j = \frac{1}{N_j}\int_{-l}^{l} U(x) P_j(x)dx. \tag{43}$$

3 Intensity Factors

The coefficients a_n are known, so that the entire perturbation stress field and the perturbation electric displacement can be obtained. However, in fracture mechanics, it is of importance to determine the perturbation stress τ_{yz} and the perturbation electric displacement D_y in the vicinity of the crack's tips. $\tau_{yz}^{(1)}$, $\tau_{yz}^{(2)}$, $\tau_{yz}^{(3)}$, $D_y^{(1)}$, $D_y^{(2)}$ and $D_y^{(3)}$ along the crack line can be expressed respectively as

$$\tau_{yz}^{(1)}(x,h) = \tau_{yz}^{(2)}(x,h) = \tau_{yz}^{(2)}(x,0) = \tau_{yz}^{(3)}(x,0) =$$
$$-\frac{c_{44}}{\pi}\sum_{n=1}^{\infty} a_n G_n \int_0^{\infty} [1 + e^{-sh}] J_{2n-1}(sl)\cos(xs)ds, \tag{44}$$
$$D_y^{(1)}(x,h) = D_y^{(2)}(x,h) = D_y^{(2)}(x,0) = D_y^{(3)}(x,0) =$$
$$-\frac{e_{15}}{\pi}\sum_{n=1}^{\infty} a_n G_n \int_0^{\infty} [1 + e^{-sh}] J_{2n-1}(sl)\cos(xs)ds. \tag{45}$$

An examination of Eqs.(44) and (45), the singular part of the stress field and the singular part of the electric displacement can be obtained respectively from the relationship[25]

$$\int_0^{\infty} J_n(sa)\cos(bs)ds = \begin{cases} \dfrac{\cos[n\arcsin(b/a)]}{\sqrt{a^2 - b^2}}, & a > b, \\ -\dfrac{a^n \sin(n\pi/2)}{\sqrt{b^2 - a^2}[b + \sqrt{b^2 - a^2}]^n}, & b > a. \end{cases} \tag{46}$$

The singular part of the stress field and the singular part of the electric displacement can be expressed respectively as follows ($l < x$) :

$$\tau = \frac{c_{44}}{\pi}\sum_{n=1}^{\infty} a_n G_n H_n(x), \tag{47}$$

$$D = \frac{e_{15}}{\pi}\sum_{n=1}^{\infty} a_n G_n H_n(x), \tag{48}$$

where $H_n(x) = \dfrac{(-1)^{n-1} l^{2n-1}}{\sqrt{x^2 - l^2}[x + \sqrt{x^2 - l^2}]^{2n-1}}$.

We obtain the stress intensity factor K as

$$K = \lim_{x \to l^+} \sqrt{2\pi(x - l)} \cdot \tau = \frac{2c_{44}}{\sqrt{l}}\sum_{n=0}^{\infty} a_n \frac{\Gamma(2n - 1/2)}{(2n - 2)!}. \tag{49}$$

We obtain the electric displacement intensity factor D_L as

$$D_L = \lim_{x \to l^+} \sqrt{2\pi(x - l)} \cdot D = \frac{2e_{15}}{\sqrt{l}}\sum_{n=0}^{\infty} a_n \frac{\Gamma(2n - 1/2)}{(2n - 2)!} = \frac{e_{15}}{c_{44}}K. \tag{50}$$

4 Numerical Calculations and Discussion

From the works[27-34], it can be seen that the Schmidt method is performed satisfactorily if the first ten terms of the infinite series (39) are obtained. The stress intensity factor K and the electric displacement intensity factor D_L are calculated numerically. The results of the present paper are shown in Figs.2 to 7. From the results, the following observations are very significant:

(i) The stress and the electric displacement intensity factors depend on the crack length and the distance between two parallel cracks.

(ii) The stress and the electric displacement intensity factors of the two parallel cracks decrease when the distance between cracks decreases. However, the stress and the electric displacement intensity factors of the two parallel cracks decrease when the length of cracks increases. This phenomenon is called crack shielding effect.

(iii) The electric displacement intensity factors for the permeable crack surface conditions are much smaller than the results for the impermeable crack surface conditions as shown in Fig.3, Fig.5, Fig.7 and in Eq.(50).

(iv) The stress intensity factor does not depend on the material constants. However, the electric displacement intensity factor depends on the shear modulus and the dielectric parameter.

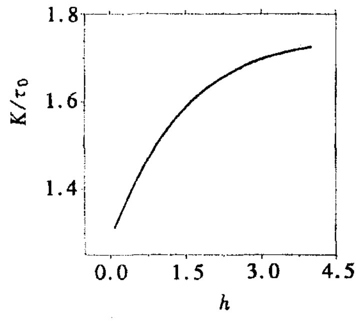

Fig.2 The stress intensity factor versus h for $l = 1.0$

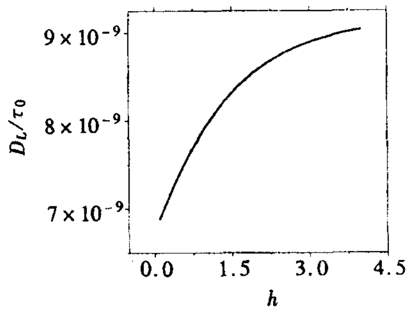

Fig.3 The electric displacement intensity factor versus h for $l = 1.0$

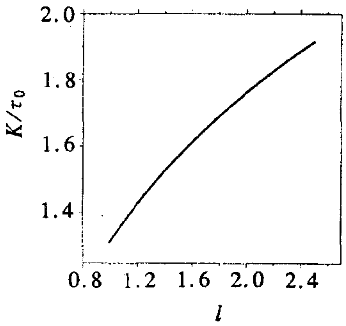

Fig.4 The stress intensity factor versus l for $h = 0.1$

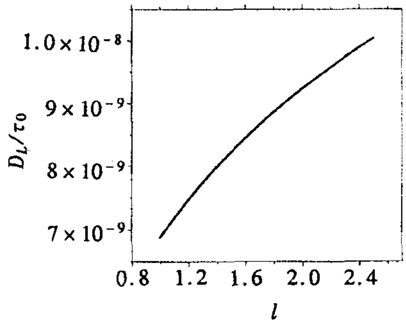

Fig.5 The electric displacement intensity factor versus l for $h = 0.1$

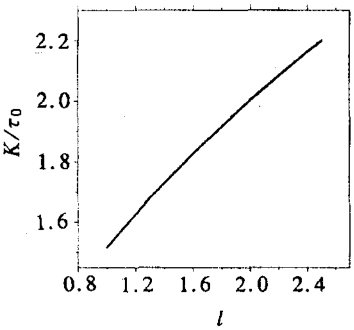

Fig.6 The stress intensity factor versus l for $h = 1.0$

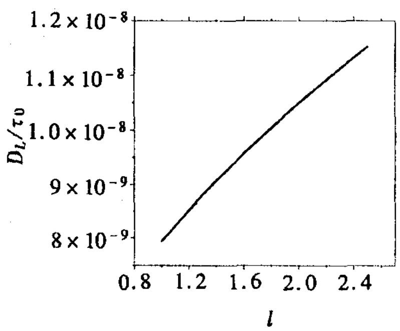

Fig.7 The electric displacement intensity factor versus l for $h = 1.0$

References:

[1] Deeg W E F. The analysis of dislocation, crack and inclusion problems in piezoelectric solids[D]. Ph D thesis. Stanford University, 1980.

[2] Pak Y E. Crack extension force in a piezoelectric material[J]. *Journal of Applied Mechanics*, 1990, **57**(4):647 - 653.

[3] Pak Y E. Linear electro-elastic fracture mechanics of piezoelectric materials[J]. *International Journal of Fracture*, 1992, **54**(1):79 - 100.

[4] Sosa H A, Pak Y E. Three-dimensional eigenfunction analysis of a crack in a piezoelectric ceramics [J]. *International Journal of Solids and Structures*, 1990:**26**(1):1 - 15.

[5] Sosa H A. Plane problems in piezoelectric media with defects[J]. *International Journal of Solids and Structures*, 1991, **28**(4):491 - 505.

[6] Sosa H. On the fracture mechanics of piezoelectric solids[J]. *International Journal of Solids and Structures*, 1992, **29**(8):2613 - 2622.

[7] Suo Z, Kuo C M, Barnett D M, *et al*. Fracture mechanics for piezoelectric ceramics[J]. *Journal of Mechanics and Physics of Solids*, 1992, **40**(5):739 - 765.

[8] Pack S B, Sun C T. Fracture criteria for piezoelectric ceramics[J]. *Journal of American Ceramics Society*, 1995, **78**(7):1475 - 1480.

[9] Zhang T Y, Tong P. Fracture mechanics for a mode Ⅲ crack in a piezoelectric material[J]. *International Journal of Solids and Structures*, 1996, **33**(5):343 - 359.

[10] Gao H, Zhang T Y, Tong P. Local and global energy rates for an elastically yielded crack in piezoelectric ceramics[J]. *Journal of Mechanics and Physics of Solids*, 1997, **45**(4):491 - 510.

[11] WANG Biao. Three dimensional analysis of a flat elliptical crack in a piezoelectric material[J]. *International Journal of Engineering Science*, 1992, **30**(6):781 - 791.

[12] Narita K, Shindo Y. Scattering of Love waves by a surface-breaking crack in piezoelectric layered media[J]. *JSME International Journal, Series A*, 1998, **41**(1):40 - 52.

[13] Narita K, Shindo Y. Scattering of anti-plane shear waves by a finite crack in piezoelectric laminates [J]. *Acta Mechanica*, 1999, **134**(1):27 - 43.

[14] Zhou Z G, Wang B, Cao M S. Analysis of two collinear cracks in a piezoelectric layer bonded to dissimilar half spaces subjected to anti-plane shear[J]. *European Journal of Mechanics, A/Solids*, 2001, **20**(2):213 - 226.

[15] Yu S W, Chen Z T. Transient response of a cracked infinite piezoelectric strip under anti-plane impact[J]. *Fatigue and Engineering Materials and Structures*, 1998, **21**(4):1381 - 1388.

[16] Zhang T Y, Hack J E. Mode-Ⅲ cracks in piezoelectric materials[J]. *Journal of Applied Physics*, 1992, **71**(9):5865 - 5870.

[17] McMeeking R M. On mechanical stress at cracks in dielectrics with application to dielectric breakdown[J]. *Journal of Applied Physics*, 1989, **62**(11):3116 - 3122.

[18] Suo Z. Models for breakdown-resistant dielectric and ferroelectric ceramics[J]. *Journal of the Mechanics and Physics of Solids*, 1993, **41**(6):1155 - 1176.

[19] Dunn M L. The effects of crack face boundary conditions on the fracture mechanics of piezoelectric solids[J]. *Engineering Fracture of Mechanics*, 1994, **48**(1):25 - 39.

[20] Zhang T Y, Tong P. Fracture mechanics for a mode Ⅲ crack in a piezoelectric material[J]. *International Journal of Solids and Structures*, 1996, **33**(5):343 - 359.

[21] Sosa H, Khutoryansky N. Transient dynamic response of piezoelectric bodies subjected to internal electric impulses[J]. *International Journal of Solids and Structures*, 1999, **36**(9):5467 - 5484.

[22] Soh A K, Fang D N, Lee K L. Analysis of a bi-piezoelectric ceramic layer with an interfacial crack subjected to anti-plane shear and in-plane electric loading[J]. *European Journal of Mechanics, A/Solid*, 2000, **19**(6): 961 - 977.

[23] Morse P M, Feshbach H. Methods of Theoretical Physics[M]. New York: McGraw-Hill, 1958, 1, 828 - 930.

[24] Gradshteyn I S, Ryzhik I M. *Table of Integral, Series and Products*[M]. New York: Academic Press, 1980, 1035 - 1037.

[25] Erdelyi A. *Tables of Integral Transforms*[M]. Vol **1**. New York: McGraw-Hill, 1954, 34 - 89.

[26] Amemiya A, Taguchi T. *Numerical Analysis and Fortran*[M]. Tokyo: Maruzen, 1969, 105 - 123.

[27] Itou S. Three dimensional waves propagation in a cracked elastic solid[J]. *Journal of Applied Mechanics*, 1978, **45**(5): 807 - 811.

[28] Zhou Z G, Bai Y Y, Zhang X W. Scattering of harmonic shear waves by a finite crack by using the non-local theory[J]. *International Journal of Engineering Science*, 1999, **37**(5): 609 - 620.

[29] Zhou Z G, Bai Y Y, Zhang X W. Two collinear Griffith cracks subjected to uniform tension in infinitely long strip[J]. *International Journal of Solids and Structures*, 1999, **36**(36): 5597 - 5609.

[30] Zhou Z G, Han J C, Du S Y. Investigation of a Griffith crack subject to anti-plane shear by using the non-local theory[J]. *International Journal of Solids and Structures*, 1999, **36**(26): 3891 - 3901.

[31] Zhou Z G, Wang B, Du S Y. Scattering of harmonic anti-plane shear waves by a finite crack by using the non-local theory[J]. *International Journal of Fracture*, 1998, **91**(1): 13 - 22.

[32] Zhou Z G, Zhang X W, Bai Y Y. Investigation of two Griffith cracks subject to uniform tension by using the non-local theory[J]. *International Journal of Engineering Science*, 1999, **37**(13): 1709 - 1722.

[33] Zhou Z G, Shen Y P. Investigation of the scattering of harmonic shear waves by two collinear cracks using the non-local theory[J]. *Acta Mechanica*, 1999, **135**(3): 169 - 179.

[34] ZHOU Zhen-gong, WANG Biao. Investigation of a Griffith crack subjected to uniform tension using the non-local theory by a new method [J]. *Applied Mathematics and Mechanics (English Edition)*, 1999, **20**(10): 1099 - 1107.